A statistical method for estimating speed from single loop detectors

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Abstract

Single loop inductance detectors measure traffic volume and occupancy, but not speed. However, speed is roughly equal to the ratio of volume to occupancy, multiplied by the average vehicle length. To obtain a simple conversion rule, it is often assumed that the average vehicle length is constant. We and other authors have checked the validity of this assumption and found that it does not hold in general. In this paper we present an alternative. Based on the assumption that during low occupancy vehicles travel on average at a known free flow speed, we can estimate a time dependent vehicle length curve. Using this curve, instead of a constant, we obtain an speed estimate that is approximately unbiased. We note that the estimator suffers from a large variance during low traffic consitions, but then show that this can be remedied by using an adaptive filter. We have successfully tested our method on double loop ("speed trap") traffic data from the I80 Emeryville field experiment.

1 Introduction

Single loop inductance detectors are by far the most common traffic measurement device. A loop detector measures traffic volume (the number of vehicles during a time interval) and occupancy (the fraction of time during which the presence of a vehicle is sensed). Let $N(d,t)$ denote the volume ("flow") and $\rho(d,t)$ the occupancy observed at a particular loop detector on day d during time interval t. Single loop detectors do not directly measure speed. This is unfortunate, because speed is perhaps the single most useful variable for traffic control and traveler information systems.

Let us fix a day d and a time of day t and consider the following situation. Suppose that at a given detector during a 30 second time interval N vehicles pass with (effective[1]) lengths $L_1, \dots, L_N$ and speeds $v_1, \dots, v_N$. The occupancy is given by $\rho = \sum_{i=1}^N L_i/v_i$. Now, if all speeds are equal during the sample period, $v = v_1 = \dots = v_N$, it follows that

$$\rho = \frac{1}{v}\sum_{i=1}^{N} L_i = \frac{N\bar{L}}{v}, \tag{1}$$

[1] The *effective vehicle length* is equal to the length of the vehicle plus the length of the loop's detector zone.

where $\bar{L} = \sum_{i=1}^{N} L_i/N$ is the average of the vehicle lengths. We see that if the average vehicle length is known, we can infer the common speed. We model the lengths L_i as random variables with common mean (mathematical expectation) μ. Note that the L_i and $\bar{L}$ are not directly observed. If μ were known, while the average $\bar{L}$ is not, then a sensible estimate of the common speed may be obtained by replacing the average by the mean in (1).

$$\hat{v} = \frac{N\mu}{k}. \tag{2}$$

Re-writing, we find $\hat{v} = v\mu/\bar{L}$. Since the expectation of $1/\bar{L}$ is not equal to $1/\mu$, the expectation of $\hat{v}$ is not equal to v. In other words, $\hat{v}$ is not an unbiased estimator of v, despite our assumption that all v_i are equal. If the number of vehicles N is not too small, then $\bar{L}$ should be reasonably close to its mean and the bias negligible. Henceforth, we neglect this bias issue and use formula (2) to estimate speed. We thus focus on estimating the mean vehicle length, μ.

2 Estimation of the mean vehicle length

Currently, it is a wide spread practice to take the mean vehicle length to be constant, independent of the time of day. The validity of this assumption has been examined by many authors (e.g. Hall and Persaud, 1989 and Pushkar et al., 1994), including ourselves (Jia et al., 2000) and it is now generally recognized that it does not hold up in general. This is further illustrated by double-loop data from Interstate 80 near San Francisco (Coifman et al., 2000), which allows direct measurement of speed. Figures 1 and 2 show the speed and the average (effective) vehicle length at detector station 2 in the East bound inner (fast) lane 1 and the outer lane 5. We see a clear daily trend in effective vehicle length in lane 5. We believe that this trend can be ascribed to the ratio of trucks to cars varying with the time of day. This is confirmed by the fact that the vehicle length in the fast lane 1, with negligible truck presence, is almost constant. We thus assume that the mean vehicle length depends on the time of day, denote it by μ_t to reflect this dependence, and consider how μ_t can be estimated.

Suppose we have observed $N(d,t)$ and $\rho(d,t)$ for a number of days. Let $\alpha_{0.6}$ denote the 60-th percentile of the observed occupancies. Assume that during all time intervals when $\rho(d,t) < \alpha_{0.6}$ all vehicles travel at a common speed v_{FF}. Since we may assume that any freeway is uncongested at least 60 per cent of the time, v_{FF} may be regarded as the free flow speed. Throughout this paper we assume that v_{FF} is known or estimated from exterior sources of information.

By our assumption on constant free flow speed, we have for all (d, t) such that $\rho(d, t) < \alpha_{0.6}$

$$\bar{L}(d, t) = \frac{v_{FF}\rho(d, t)}{N(d, t)}.$$

If we assume that the average vehicle length $\bar{L}(d, t)$ does not depend on whether the occupancy is above or below the threshold then

$$\mathbb{E}(\bar{L}(d, t) \mid \rho(d, t) < \alpha_{0.6}) = \mathbb{E}\bar{L}(d, t) = \mu_t.$$

For fixed t we can obtain an unbiased estimate of μ_t as

$$\hat{\mu}_t = \frac{1}{\#\{d : \rho(d, t) < \alpha_{0.6}\}} \sum_{d:\rho(d,t)<\alpha_{0.6}} \frac{v_{FF}\rho(d, t)}{N(d, t)}.$$

In Figure 3 we have plotted the time of day t versus $v_{FF}\rho(d, t)/N(d, t)$ for all times (d, t) when $\rho(d, t) < \alpha_{0.6}$. We can now estimate the expectation μ_t of the effective vehicle length by fitting a regression line to this scatter plot, via *loess* (Cleveland, 1979). The smooth regression line seen in Figure 3 is our estimator $\hat{\mu}_t$ of μ_t. Note the absence of points for times between 3pm and 6pm when I80 East is always congested.

Once we have an estimator $\hat{\mu}_t$ of μ_t, we define a (preliminary) estimator of $v(d, t)$ as

$$\hat{v}(d, t) = \frac{N(d, t)\hat{\mu}_t}{\rho(d, t)}. \tag{3}$$

This estimator is plotted in Figure 4. We see that it performs very well during heavy traffic and congestion. In particular, it exhibits little bias during the time period 3pm to 6pm over which the smoothing shown in Figure 3 was extrapolated. Unfortunately, the variance of the estimator during times of light traffic, particularly in the early hours of each day, is unacceptably large. This is clearly visible in Figure 4 with estimated speeds on day 3 around 1 am shooting up to 120 mph shortly before plummeting to 30 mph. The true speed at that time is nearly constant at 64 mph. Recall that our preliminary estimate (3) is obtained by replacing the average (effective) vehicle length $\bar{L}(d, t)$ by (an estimate of) its expectation μ_t. When only a few vehicles pass the detector during a given time interval, the average vehicle length will have a large variance. Hence, in light traffic, the average vehicle length is likely to differ substantially from the mean. For instance, if only 10 vehicles pass, then it makes a big difference if there are 6 cars and 4 trucks or 7 cars and 3 trucks. This explains the large fluctuations of our preliminary estimator $\hat{v}$ during light traffic.

3 Smoothing

Coifman (2001) suggests a simple fix for the unstable behavior of $\hat{v}$ during light traffic. He sets the estimated speed equal to the free flow speed v_{FF} when the occupancy is low.

$$\hat{v}_{\text{coifman}}(d,t) = \begin{cases} \hat{v}(d,t) & \text{if } \rho(d,t) \geq \alpha_{0.6} \\ v_{FF} & \text{otherwise .} \end{cases}$$

The performance of this estimator, in terms of mean squared error, is certainly not bad. However, about 16 out of every 24 hours (60%), the estimated speed is a constant and that is not realistic. We can do better, in appearance as well as in mean squared error.

It is clear that we need to smooth our preliminary estimate $\hat{v}(d,t)$, but only when the volume is small. For the purpose of real time traffic management, it is important that our smoother be causal and easy to compute with minimal data storage. Taking all this into consideration, we used an exponential filter with varying weights. A smoothed version $\tilde{v}$ of $\hat{v}$ is defined recursively as

$$\tilde{v}(d,t) = w(d,t)\hat{v}(d,t) + (1 - w(d,t))\tilde{v}(d,t-1), \tag{4}$$

where

$$w(d,t) = \frac{N(d,t)}{N(d,t) + C}, \tag{5}$$

and C is a smoothing parameter to be specified. If the time interval is of length 5 minutes, then a reasonable value would be $C = 50$. If the volume $N(d,t)$ approaches capacity, say $N(d,t) = 100$ vehicles per 5 minutes (1200 veh/h), then there is hardly any need for smoothing and the new observation receives substantial weight 2/3. On the other hand, if the volume is very small, say $N(d,t) = 10$ (120 veh/h), then the smoothing is quite severe with the new observation receiving a weight of only 1/6.

Our filtered estimator $\tilde{v}$ is plotted in Figure 5. The correspondence with the true speed is very good. The large variability during light traffic that plagued the preliminary estimator $\hat{v}$ has been suppressed, while its good performance during heavy traffic and congestion has been retained.

We will now explain how our filter is "inspired" by the familiar Kalman filter. Suppose that the true, unobserved speed evolves as a simple random walk:

$$v_t = v_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0, \tau^2). \tag{6}$$

Suppose we observe $\hat{v}_t = N_t \hat{\mu}_t / k_t = v_t \mu_t / \bar{L}_t$, where $\hat{\mu}_t$ is our estimate of $E\bar{L}_t = \mu_t$. We will work conditionally on the observed volume N_t. The conditional expectation of $\hat{v}_t$ is—though not quite equal—hopefully close to v_t. Using a one step Taylor approximation we find that the conditional variance of $\hat{v}_t$ is of the order $1/N_t$. This "inspires" a measurement equation

$$\hat{v}_t = v_t + \xi_t, \qquad \xi_t \sim \mathcal{N}(0, \sigma_t^2) = \mathcal{N}(0, \sigma^2/N_t). \tag{7}$$

Finally, we assume that all error terms ε_t and ξ_t are independent. Note that the variance of the measurement error ξ_t depends inversely on the observed volume N_t. In light traffic, when N_t is small the variance is large. This is exactly the problem we noted in Figure 4.

The Kalman filter recursively computes the conditional expectation of the unobserved state variable v_t given the present and past observations $\hat{v}_1, \hat{v}_2, \ldots, \hat{v}_t$.

$$\tilde{v}_t = \mathbb{E}(v_t \mid \hat{v}_1, \hat{v}_2, \ldots, \hat{v}_t).$$

In our simple model we can easily derive the Kalman recursions. They are

$$\tilde{v}_t = w_t \hat{v}_t + (1 - w_t)\tilde{v}_{t-1},$$

with

$$w_t = \frac{P_{t-1} + \tau^2}{P_{t-1} + \tau^2 + \sigma_t^2} = \frac{N_t}{N_t + \sigma^2/(P_{t-1} + \tau^2)},$$

where P_t is the prediction error $\mathbb{E}(v_t - \tilde{v}_t)^2$.

We note the correspondence of these Kalman recursions with our filter 4. We decided not to try to estimate σ^2 and τ^2 partly because we feel that that would be difficult to do reliably and partly because that would mean taking our simple model a little too seriously.

4 Remarks

4.1 Known free flow speed

We assume above that the free flow speed v_{FF} is known, which is typically not true. We believe free flow speed depends primarily on the number of lanes and on the lane number, so in practice we use a table of free flow speeds (in miles per hour) which are based on double loop data from Caltrans district 4 "Bay Area". Interestingly, the data seem to fit a very simple formula to a reasonable degree. For an n lane freeway, the freeflow speed in lane m equals 65 + 2.5*(n - m) mph.

lane number				
1	71.3	71.9	74.8	76.5
2	65.8	69.7	71.0	74.0
3		62.7	67.4	72.0
4			62.8	69.2
5				64.5

Clearly, it would be preferable to have an independent method to estimate site specific free flow speed. Petty et al.'s (1998) cross-correlation approach works well when occupancy and volume are measured in 1 second intervals. However, 20 or 30 second measurement intervals are more common and at such aggregation this method breaks down.

4.2 Further assumptions on mean vehicle length

We have assumed that the mean (expected) vehicle length μ_t depends on the time of day only. However, we have noticed that μ_t also depends on

1. *Day of the week.* The vehicle mix on a Monday differs from a Sunday.

2. *Lane.* There is a higher fraction of trucks in the outer lanes.

3. *Location of the detector station.* Certain routes are more heavily traveled by trucks than others.

4. *Detector sensitivity.* Loop detectors are fairly crude instruments that are almost impossible to calibrate accurately. If a detector is not properly calibrated, the occupancy measurements will be biased.

To account for all this, we must form separate estimates of μ_t to cover these different situations. We store estimates of μ_t for every 5 minute interval, for every day of the week and for every lane at every detector station. In real time, the appropriate values are retrieved, multiplied by the observed volume to occupancy ratio and filtered.

4.3 Other methods

We briefly review two other methods that also do not assume a fixed value for $\bar{L}(d,t)$, beginning with a method of Coifman (2001) which has been modified by Jia et al. (2002). Suppose that we have a zero-one valued state variable $S(d,t)$ which indicates congestion or free flow. The state variable could be defined, for instance, by thresholding the occupancy $\rho(d,t)$. While the state is "free flow", the algorithm tracks $\bar{L}(d,t)$, assuming constant free flow speed. As soon as the state becomes "congested", $\bar{L}(d,t)$ is kept fixed and the speed $v(d,t)$ is tracked.

The main problem we experienced with this algorithm is that it depends crucially on the state $S(d,t)$. In particular, if the state is believed to be free flow, while congestion has already set in, the method goes badly astray. We found it difficult to develop a good rule to define $S(d,t)$. In fact, this difficulty was the main reason for us to look for a different approach.

Building on work of Dailey (1999), Wang and Nihan (2000) propose a model based approach to estimate $\bar{L}(d,t)$ and $v(d,t)$. Their log linear model relates $\bar{L}(d,t)$ to the expectation and variance of the occupancy $\rho(d,t)$, to the volume $N(d,t)$ and to two indicator functions that distinguish between high flow and low flow periods. The model has five parameters which need to be estimated from double loop data. It is not at all clear if these parameter estimates carry over to a particular, single loop location of interest. Wang and Nihan (2000) defer this issue to future research.

4.4 Estimating truck fraction

If we have a speed estimate $\hat{v}(d,t)$, we can easily obtain an estimate of the average (effective) vehicle length $\bar{L}(d,t)$ as

$$\hat{L}(d,t) = \frac{\hat{v}(d,t)\rho(d,t)}{N(d,t)}.$$

For certain applications it is of interest to determine the truck volume on a particular freeway segment. Specialized equipment to count trucks is costly and manual counting is only feasible for short time periods and a limited number of locations. Kwon et al. (2003) suggest how truck volume can be estimated routinely from the estimated effective vehicle length. Denote by $\bar{L}_c(d,t)$ and $\bar{L}_t(d,t)$ the average effective length of the cars and trucks that passed on day d at time t. We have

$$\bar{L}(d,t) = p(d,t)\bar{L}_t(d,t) + (1 - p(d,t))\bar{L}_c(d,t),$$

where $p(d,t)$ is the proportion of trucks. Let ℓ_c and ℓ_t denote the mean effective length of cars and trucks. These means are largely determined by which vehicles are considered to be trucks. Replacing averages by their means, we can estimate the truck proportion as

$$\hat{p}(d,t) = \frac{\hat{L}(d,t) - \ell_c}{\ell_t - \ell_c}.$$

We estimate the truck volume by $\hat{p}(d,t)N(d,t)$.

Loop detectors record volume and occupancy by thresholding an electrical signal. The threshold has to be set manually, and there is no practical method for precise calibration. As a result, any two detectors will likely record different occupancies for the same vehicle traveling at constant speed. Translated to effective vehicle length, this difference can be several feet. Our speed estimator is designed to be insensitive to this problem, but the above truck volume estimator is not. The possible effect of detector bias on the estimator can be gauged by perturbing $\hat{L}(d,t)$.

4.5 The PeMS project

Our efforts described here are part of the Performance and Measurement System (PeMS) project. This project is a collaboration of the California Department of Transportation (Caltrans) and the University of California at Berkeley. The core of PeMS is a database which receives and stores 30 second loop detector measurements from the entire state. Since most of the detectors in California are in single loop configuration, we have a pressing need for a good method to obtain speeds. Once occupancy, volume and speed are available, PeMS proceeds to compute such performance measures as VMT (vehicle miles traveled), VHT (vehicle hours traveled) and travel time on selected routes. PeMS is soon to be officially deployed by Caltrans.

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References

W.S. Cleveland, Robust locally weighted regression and smoothing scatter plots. *Journal of the American Statistical Association* **74** 829–836, 1979.

B.A. Coifman, D Lyddy and A. Skabardonis, The Berkeley Highway Laboratory- Building on the I-880 Field Experiment. In *Proc. IEEE ITS Council Annual Meeting* 5–10, 2000.

B.A. Coifman, Improved velocity estimation using single loop detectors. *Transportation Research A* **35** 863–880, 2001.

D.J. Dailey, A statistical algorithm for estimating speed from single loop volume and occupancy measurements. *Transportation Research B* **33** 313–322, 1999.

F.L. Hall and B.N. Persaud, Evaluation of speed estimates made with single-detector data from freeway traffic management systems, *Transportation Research Record* **1232**, 9–16, 1989.

Z. Jia, C. Chen, B.A. Coifman, P.P. Varaiya, The PeMS algorithms for accurate, real-time estimates of g-factors and speeds from single-loop detectors. Presented at IEEE 4th International ITS Conference, 2001.

J. Kwon, P. Varaiya, P. and A. Skabardonis, Estimation of Truck Traffic Volume from Single Loop Detector Outputs Using Lane-to-lane Speed Correlation, Presented in TRB 2003; Accepted for publication in TRB 2003.

K.F. Petty, P.J. Bickel, J. Jiang, M. Ostland, J.A. Rice, Y. Ritov, and F. Schoenberg, Accurate estimation of travel times from single-loop detectors, *Transportation Research A* **32**, 1–17, 1998.

A. Pushkar, F.L. Hall and J.A. Acha-Daza, Estimation of speeds from single-loop freeway flow and occupancy data using cusp catastrophe theory model. *Transportation Research Record* **1457**, 149–157, 1994.

Y. Wang and N. Nihan, Freeway traffic speed estimation with single loop outputs. *Transportation Research Record* **1727**, 120–126, 2000.

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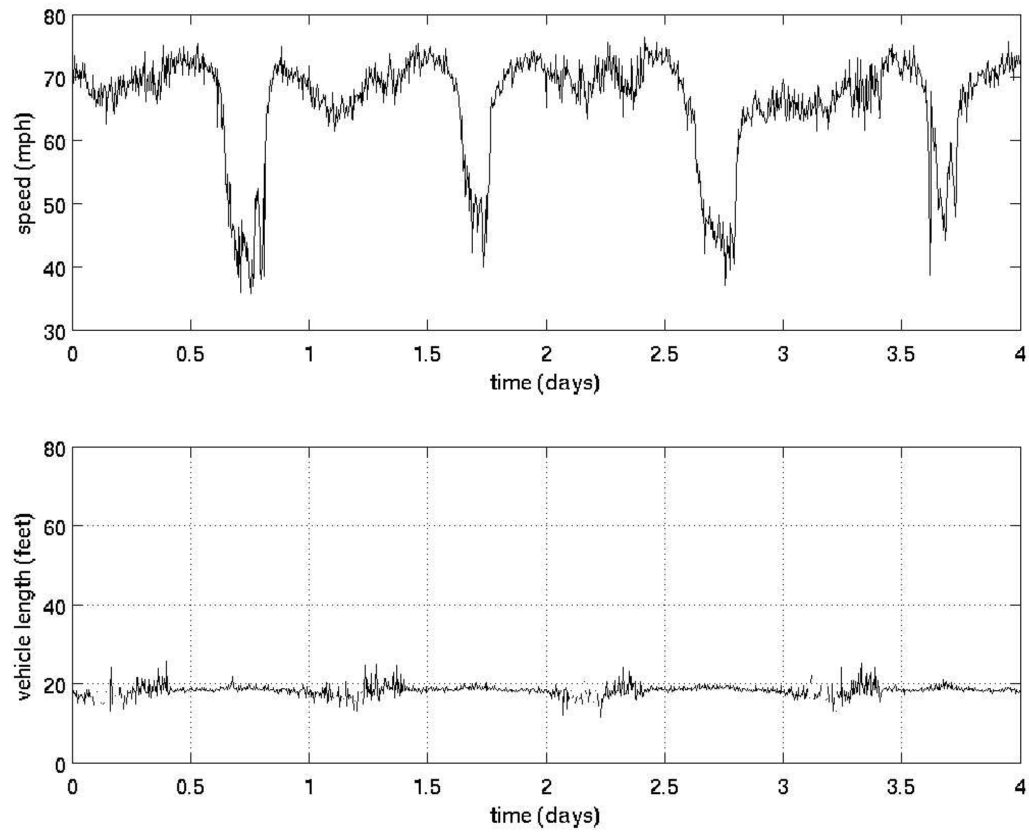

Figure 1: speed (top) and effective vehicle length (bottom) in the fast lane for four weekdays on I80.

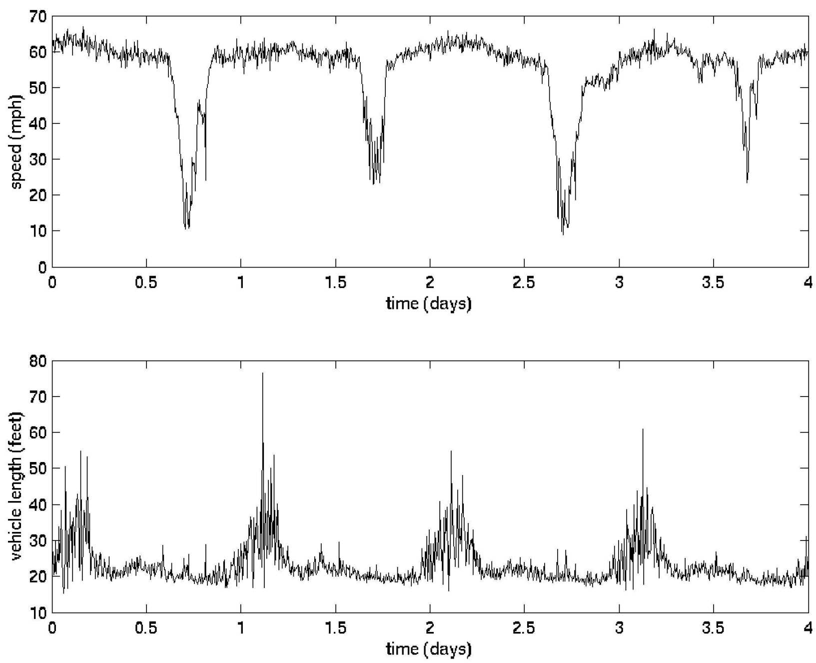

Figure 2: speed (top) and effective vehicle length (bottom) in the truck lane for four weekdays on I80.

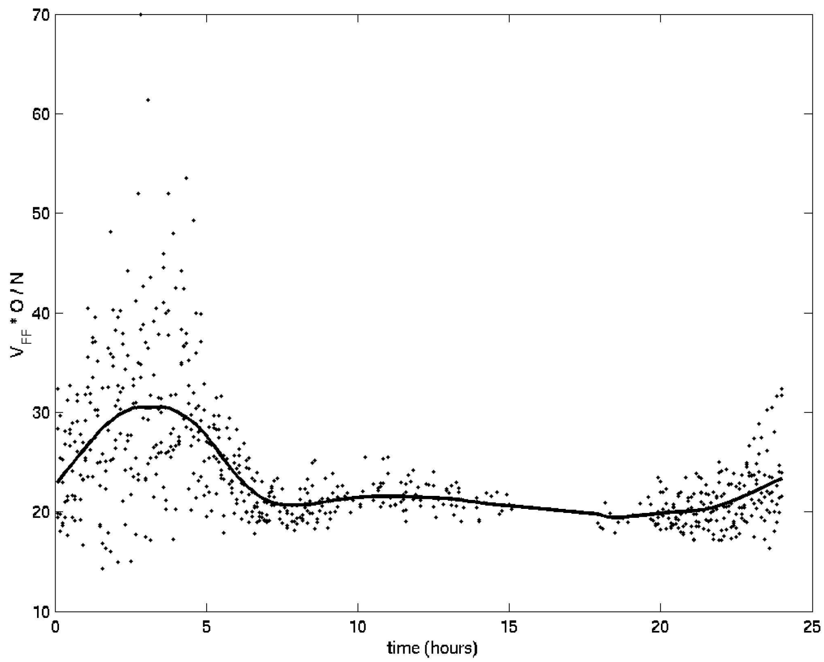

Figure 3: Estimating the mean effective vehicle length μ_t.

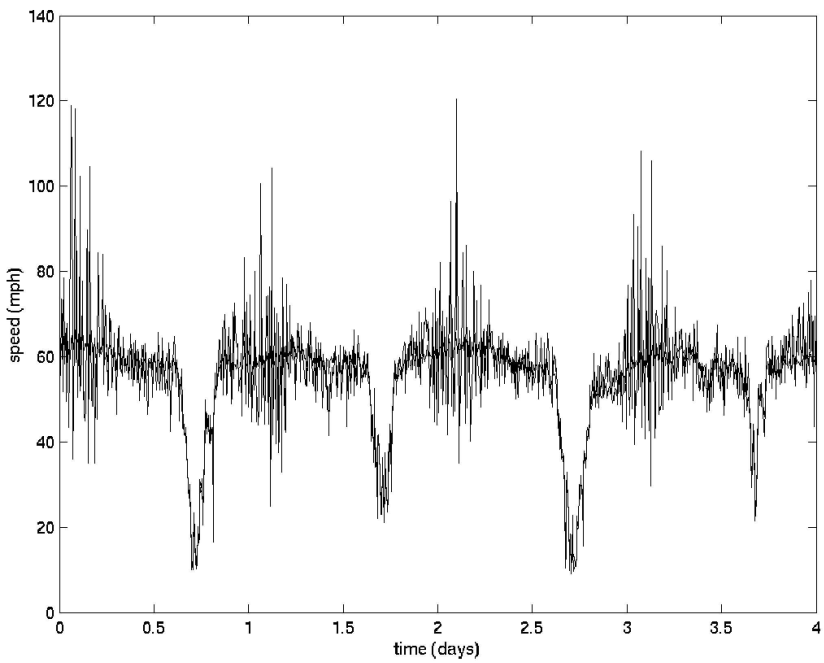

Figure 4: Our preliminary estimate, defined in (3), superimposed on the true speed. Note the large variance, especially during the early hours of the day. Note also, that during congestion the estimator performs very well.

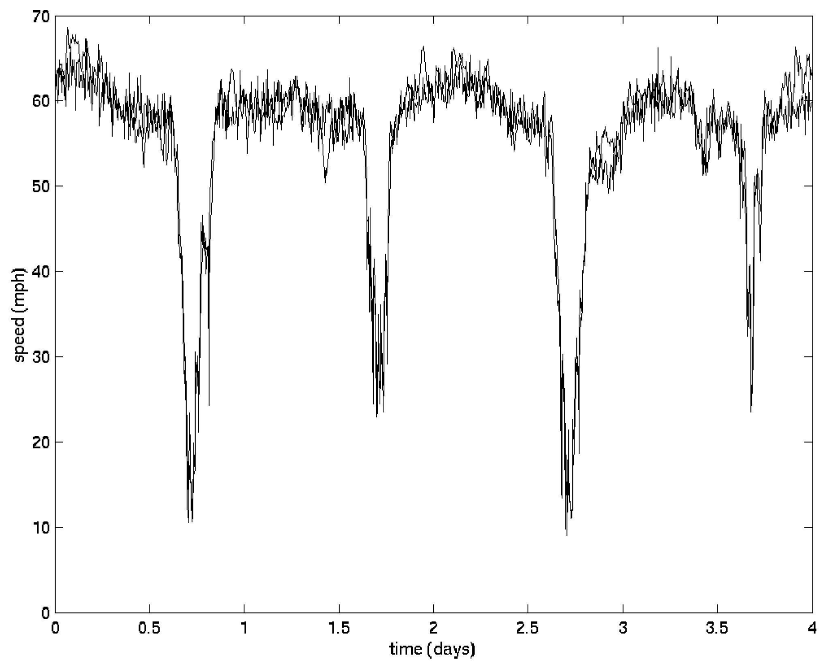

Figure 5: Our estimate $\tilde{V}$, defined in (4), superimposed on the true speed. The estimate shows good correspondence with the true speed.